

## **IPM component 3 – STEM**

### **Week 7: Paragraph building**

#### **Introduction**

In the previous set of notes we focused on how to recognise and write topic sentences. Part of these notes included how to write a second or third sentence which continued on from the topic sentence. This forms part of what is called paragraph building which is what we now look at in these notes.

#### **Paragraph building**

##### *Developing the first paragraph*

When one writes a topic sentence one tends to know how one wants to develop the idea of topic sentence. This development becomes the rest of the paragraph. For example, if my topic sentence is based on the statistical idea of lines of best fit I might write:

Many courses utilise the notion of least squares line of best fit.

I know what it is about least squares line of best fit I want to develop in more detail. It could be

- *an alternative approach to the standard way of deriving the relevant equations:*

“Many courses utilise the notion of least squares line of best fit. The usual way of deriving the least squares line is straight forward for students who have a basic knowledge of calculus. Here we present a simple analytic geometry approach as an alternative way of deriving these equation.”

or

- *an alternative to basing the analysis on the mean as the measure of center:*

“Many courses utilise the notion of least squares line of best fit. This is based on the idea of deviations from the mean data value. But this approach makes the line of best fit sensitive to outliers. In this article we show how basing the least squares analysis on the median produces a line of best fit which is more robust.”

or

- *a general statement about the idea behind lines of best fit:*

“Many courses utilise the notion of least squares line of best fit. The method is an algebraic technique for fitting linear equations to experimental data. Since experimental data never all lie on a single straight line the aim of the technique is to find the “best” straight line which passes through the data.”

or

- *a historical account of how least squares analysis developed:*

“Many courses utilise the notion of least squares line of best fit. The first clear and concise account of the method of least squares was by Legendre in 1805. Within ten years after Legendre’s publication the method of least squares was adopted as a standard tool in astronomy in France and Italy.”

or

- *a definition of least squares:*

“Many courses utilise the notion of least squares line of best fit. The line of best fit is one which passes through the mean of the data and minimises the distance between the line and all other data points. It also optimises the gradient of the line”

or

- *a critique:*

“Many courses utilise the notion of least squares line of best fit. The standard approach to least squares analysis uses the mean as its basis. However, this is known to not be robust since the analysis based on the mean is sensitive to outlying values. As a result many authors have adopted the use of ...”

So, do you want to do any of the above, or something else such as explaining, summarising, changing the focus, etc.?

### Changing the focus of the previous paragraph

If you want to change the focus from one paragraph to the next you can use the topic sentence as a transition. Used in this way different topic sentences can guide the reader logically through the separate scientific aspects of your argument. Such topic sentences can then be thought of as “bridges” between the main idea of the previous paragraph and the main idea of the next paragraph. Using transitional language allows one to make such changes in focus or theme between paragraphs. For example:

“In the previous paragraph we spoke about the advantages of the mean as used in statistics. Here we will focus on the advantages of the median. ...”

or

“Although the mean has many advantages in statistics, as mentioned in the previous paragraph, the median has certain advantages which compensate for the problems caused by the use of the mean.”

or

“Despite the advantages of basing least squares analysis on the mean the drawbacks of this are significant. Hence it is sometimes appropriate to base our analysis on the median, as can be seen by ...”

or (on a completely different topic)

“Although the ellipsoid method is a powerful tool in proving polynomial solvability, it is not used in practice because of its poor average performance. A primal simplex method is an alternative promising approach in solving LP(k).”

Topic sentences of subsequent paragraphs can also establish a connection between paragraphs separated far apart. To see this consider the text below which comes from an extended essay I myself am writing. There is a point in this text where I refer back to a comment I made much earlier in the text:

- Can you identify the language I use in order to make this reference?
- Is the sentence containing this reference a topic sentence? If it is, explain why it is a topic sentence. If not, how would you make it a topic sentence?

Suppose you are an A-level student who has just sat his exams in June. You are going to start a mathematics degree in October, just four months away. You know everything about the mathematics you have been taught. For example, you know about functions such as  $e^x$ ,  $\ln x$  and  $\sin x$ . You know these to be curves and you know how to graph them. You know what happens to them as  $x$  approaches plus infinity or minus infinity simply by looking at the behaviour of the curves at the extreme ends of the  $x$ -axes.

In terms of the definition of trig functions you know these to be the ratio of relevant sides of a right-triangle. As for  $e^x$  some of you may have seen the expression  $\lim_{n \rightarrow \infty} (1 + x/n)^{n/x}$  but you consider this to simply be an alternative way of writing  $e^x$ . And as for  $\ln x$ , you see this as the inverse of  $e^x$  so there is no need to define  $\ln x$ .

But what if I told you that this is not the case. What if I told you that you don't know the real definition of the trig functions? what if I told you that there is indeed an actual definition for  $e^x$ , and  $\ln x$ ? And this is where a transition is about to take place in your understanding of the

nature of mathematics as you start your degree. The truth is that the trig functions are not defined as you have been taught at school. For example, one definition (of several) for  $\sin x$  is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

Also,  $e^x$  and  $\ln x$  have their own definitions:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\ln x = \int_1^x \frac{1}{t} dt$$

Even  $\pi$  is defined such that this gives the number we know of:

$$\pi = 2 \int_{-1}^1 \sqrt{1-x^2} dx.$$

The first two equations you may have seen before. They are the Maclaurin expansions of  $\sin x$  and  $e^x$  respectively.

Now, you may wonder what the difference is between saying  $\sin x = \text{opposite/hypotenuse}$  and saying  $\sin x$  is given/defined by its Maclaurin series, or between saying  $\pi = \text{circumference} \div \text{diameter}$  and saying that it is the integral above. It is in these differing definitions that the transition between school mathematics and university mathematics lies.

For example, the right-hand side of the equation involving  $e^x$  is not simply a different way of “writing”  $e^x$ . The right-hand side actually *defines*  $e^x$ . This means that  $e^x$  *exists* or is *constructed* as a function because of the right-hand side, and not because  $e^x$  happens to be “ $e^x$ ”. The same applies to the last two equations. The fact is that  $\ln x$  and  $\pi$  are defined by the right-hand side of these equations. And more than this, there are multiple versions of the definition for  $e^x$ ,  $\sin x$ , and  $\ln x$ .

Returning to what you were taught at school, you know about the concept of differentiation and integration. But what if I told you that, although you know the definition of the derivative and the integral to be

$$\frac{df}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

and

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \delta x$$

you don't know the basis for these definitions. Your current understanding of the concept of differentiation is that it relates to sequences of secants through two points on a curve, such that these two points become infinitely close so as to ultimately produce a tangent at one point. As for integration, you know this to be related to the sum of areas of an infinite number of infinitely thin rectangles under a curve.

But in the world of rigorous mathematics (which is the world you are now entering) the way of thinking about the concept of differentiation and integration as being based on secants, curves, tangent lines, rectangles, and the idea of "infinitely close" is no longer used. It hasn't been used for at least 100 years. This type of thinking (the type you were taught at school) is what we call geometric thinking. However, modern mathematics (the type taught at university and used at a professional level) is founded on arithmetic. Numbers and arithmetic.

### Summary

Three possible uses of topic sentences for the second and subsequent paragraphs of a section include

- establishing a continuation from the main idea of a previous paragraph,
- acting as transitions to a new idea by referring to disadvantages, drawbacks, limitations, etc. of the main idea of a previous paragraph,
- establishing a connection between paragraphs separated far apart.

There are other possible uses for topic sentences, these being seen by reading papers from your own discipline.

### Transition words

Below is a list of some transition words that can be used to signal a change in focus from one paragraph to the next. Such transition words help to highlight changes, contrasts, or comparisons between paragraphs. This list is not exhaustive. Use relevant language references to find other transition words or phrases.

on the contrary	nevertheless	in spite of	however
on the one hand	on the other hand	rather	conversely
at the same time	nevertheless	alternatively	while/whilst
nevertheless	while this may be true		

Example: Statistics

Consider the following text taken from [9]. Again, we will analyse this text for its topic sentences and paragraph building.

1       The *ASA Statement on P-Values and Statistical Significance* stopped just short of  
2       recommending that declarations of “statistical significance” be abandoned. We  
3       take that step here. We conclude, based on our review of the articles in this special  
4       issue and the broader literature, that it is time to stop using the term “statistically  
5       significant” entirely.  
6       [...]  
7       Regardless of whether it was ever useful, a declaration of “statistical significance”  
8       has today become meaningless. Made broadly known by Fisher’s use of the phrase  
9       (1925), Edgeworth’s (1885) original intention for statistical significance was  
10      simply as a tool to indicate when a result warrants further scrutiny. But that idea  
11      has been irretrievably lost. Statistical significance was never meant to imply  
12      scientific importance, and the confusion of the two was decried soon after its  
13      widespread use (Boring 1919).

Analysis

Sentence 1 (lines 1-2): Although this sentence is quite specific (the abandonment of the idea of statistical significance) it can be considered as a topic sentence because more detail or focus can be provided as to the idea of statistical significance and/or why it should be abandoned. Hence, I would think the next sentence or the rest of the paragraph to refer to something about statistical significance and/or its abandonment.

Sentence 2 and 3 (lines 2-5). Here the idea of sentence 1 is indeed continued by the fact that the authors wish to take the step ASA did not take, i.e. abandoning the idea of statistical significance. This is expressed as “We take that step here. We conclude ...”

Also, notice that lines 3-4 illustrate the stance the authors are taking, namely that they believe the term “statistically significant” should no longer be used.

Sentence 4 (lines 7-8) start a new paragraph. Will the authors change their focus from the first paragraph or will they continue discussing the same idea of the first paragraph? Reading lines 9-13 shows us that they change the focus by now describing what the original meaning of statistical significance was supposed to be. How do they achieve this shift in focus? They do this via the phrasing “Regardless of whether it was ever useful ...” of sentence 4.

The phrasing “Regardless of whether it was ever useful ...” deserves further remark. Nowhere in the previous paragraph do the authors refer to the usefulness of statistical significance. However, this idea that statistical significance is useful can be inferred from the way it is used and taught in the discipline of statistics (any 1<sup>st</sup> year undergraduate book on statistics include statistical significance as a major section). Remember that the text above comes from a journal paper, this being aimed at an expert audience. So, the authors know that their readers are aware of the common usage of statistical significance. So, the phrasing “Regardless of whether it was ever useful ...” actually refers back not to any previous paragraph of their paper but to the general usage in statistics.

Example: Big data

Consider the following text adapted from [7]. Again, I will analyse this text for its topic sentence and paragraph building.

1 Big data has become a buzzword in the past decade, both in science and among the  
2 general public. Scientists from all areas encounter this notion in the shift of content  
3 and methods in their research, as well as in current scientific funding programs.  
4 For example, big data is one of the selected routes in the Dutch National Scientific  
5 Agenda (NWA), and the large funding program Commit2Data has been launched in  
6 the Dutch Digital Delta in 2016. [...]

7 The role of mathematics is easy to overlook and not fully recognized  
8 because technological advances are much more visible than mathematical  
9 advances, even though the latter often have more effect. Here is a small illustration.  
10 It is common knowledge that the acceleration of computers caused by  
11 technological advances follows Moore’s law: doubling of speed every 18 months.  
12 However, it is much less known that the acceleration caused by advances in  
13 mathematical methods in scientific computing and optimization is at least of the

14 same order of magnitude, and in some areas even much higher (Bixby 2012,  
15 Schilders 2008)

### Analysis

Sentence 1 (lines 1-2) is a topic sentence. It reads as a general statement about the nature of the term “big data” in certain areas.

Sentence 2 (lines 2-3) could also be considered a topic sentence, provided a minor edit in linguistic reference were made (what edit?). In this case, sentence 2 continues addressing the idea of big data by providing referring to how the areas of research, methods and funding have changed to accommodate big data.

Sentence 3 (lines 4-6) continues addressing the focus of sentence 2, this time by giving a specific example of the importance of big data in the national scientific agenda of the Netherlands.

Sentence 4 (lines 7-9) starts with a new topic sentence which has a completely different focus to that of the first paragraph, namely mathematics and technological advances. This sentence does not link to anything described in the previous paragraph. It does not even act as a transition sentence. My personal opinion is that this topic sentence is very jarring because of this lack of transition phrasing.

However, it is clear that big data analysis involves mathematics so changing the focus to mathematics and technology is relevant. In order to achieve this change a transitional word or phrase could have been used. For example, the sentence shown in red below could have been included, as well as the use of the terms shown in red on the second and fourth lines below.

**With big data comes the matter of mathematical analysis and computing power.**

The role of mathematics **in big data analysis** is easy to overlook and not fully recognized because technological advances are much more visible than mathematical advances, even though the latter often have more effect. **Furthermore**, it is common knowledge that the acceleration of computers caused by technological advances follows Moore’s law: doubling of speed every 18 months.

The last sentence (line 11-14) makes sense in the context of this paragraph since it acts as a contrasting statement to that of the previous sentence (line 9-11). This last sentence therefore continues carrying the theme of the previous sentence as well as the second paragraph as a whole.

Example: Mathematics (Complex numbers)

Consider the paragraph below.

1        Recall that real numbers can be represented geometrically as points on a number line.  
2        Similarly, complex numbers can be represented geometrically. But because they have  
3        two components, a real part and an imaginary part, we draw complex numbers as points  
4        on a two-dimensional graph. We therefore represent complex numbers on a graph by  
5        plotting the real part on a horizontal *Re* axis, and the imaginary part on a vertical *Im* axis.  
6        Having plotted a complex number  $z = x + iy$  we are now in a position to derive two new  
7        features of  $z$ : its length  $r$ , calculated via Pythagoras' theorem, and the angle  $\theta$  it makes  
8        with the real axis, calculated in terms of arctan.

Analysis

Sentence 1 (line 1) acts as a topic sentence and uses the transition word "Recall" to link back to what is a common conception of real numbers. Since this topic sentence is very broad I would now expect any one of the following concepts to be continued in subsequent sentences: "real numbers", "geometry", "points", and/or "number line".

Sentence 2 (line 2) does indeed continue an idea of sentence 1, namely that numbers can be represented geometrically. However, since real numbers and complex numbers are two different types of numbers we need a suitable transition word which differentiates these two types of numbers but still allows us to refer to them geometrically. This word is "Similarly".

Sentence 3 (line 2-4) also continues to focus on geometry of points. However, given the phrase "But because they have" I would expect to see an alternative aspect of the geometry of numbers being discussed (with particular reference to complex numbers as opposed to real numbers), which is indeed the case.

Sentence 4 (lines 4-5) continues the idea of sentence 3 by focusing on how to plot complex numbers in a geometric setting ("two-dimensional graph").

Sentence 5 (lines 6-8) contains a transition. This is seen by the use of "Having plotted ... we are now in a position to ...". This implies a change in the mathematical focus of this paragraph. (technically this should therefore be the first sentence of a new paragraph). Here we move away from the idea of geometry towards the idea of the algebraic properties of complex numbers.

### Example: Geology

Consider the following text taken from [6]. Again, I will analyse this text for its topic sentence and paragraph building.

1        Magnetic reconnection is a process that occurs throughout the universe in ionized  
2        gases (plasmas) containing embedded magnetic fields. This process converts  
3        magnetic energy to electron and ion energy, causing phenomena such as solar  
4        flares and auroras. The NASA Magnetospheric Multiscale mission has shown that  
5        in magnetic reconnection regions there are intense electric field oscillations or  
6        waves and that electrons form crescent and beam-like populations propagating  
7        both along and perpendicular to the magnetic field. This study shows that the  
8        observed electron populations are responsible for high-frequency waves including  
9        their propagation directions and frequency ranges.

### Analysis

Sentence 1 (lines 1-2) is a topic sentence. It reads as general information where the authors make a statement about where magnetic reconnections can be found. I would expect to see the next sentence continue the idea of the process of magnetic reconnection or the idea of ionized gases containing embedded magnetic fields.

Sentence 2 (lines 2-4) does indeed continue one of the ideas stated in sentence 1, namely the process of magnetic reconnection. This can be seen by the use of the words "This process".

Sentence 3 (lines 4-7) seems to start with some new aspect (i.e. the NASA mission) but if we read on we see that it in fact continues with the idea magnetic reconnection, this time focus in more detail on the process of magnetic reconnection.

Sentence 4 (lines 7-9) then continues with one of the points of detail mentioned in sentence 3.

### Example: Physics

Consider the following text taken from [8]. Again I will analyse this text for its topic sentence and paragraph building.

1        One of the interesting problems concerning experimental results is what happens  
2        when an experiment gives a null result, when the phenomenon expected is not  
3        observed. Is it because the experimental apparatus and the associated analysis  
4        procedures cannot detect or measure the phenomenon in question or is it because  
5        the phenomenon is not present? This is a real problem in the practice of physics. In

6 the Michelson-Morley experiment, one of the most famous experiments in modern  
7 physics, the experimenters expected to detect a fringe shift caused by the motion  
8 of the earth relative to the ether. They found no such fringe shift. Was it because  
9 the apparatus was faulty or because the earth's velocity relative to the ether was  
10 zero?

### Analysis

Sentence 1 (lines 1-3) is a topic sentence. It reads as a general statement about null or negative results about experiments in general.

Sentence 2 (lines 3-5) continues the theme or idea of negative results by asking why these occur.

Sentence 3 (lines 5) is a stance comment. It describes an opinion.

Sentence 4 (lines 5-8) the theme of sentence 1 is continued via a particular example of a negative result in experimental physics.

Sentence 5 (line 8-9) then picks up on the issue of the negative result described in sentence 4 by asking certain questions about that results.

### **Exercises**

#### Exercise 1

The text below come from p118 of *A concise course in advanced level statistics, with worked examples (Fourth edition)*, J. Crawshaw and J. Chambers (Nelson Thornes). Perform a similar analysis on this text as the one shown in the example above.

1 Consider the simplest type of regression function, where  $y = f(x)$  is a straight line.  
2 If the points on the scatter diagram appear to lie near a straight line, called a  
3 **regression line**, you would say that there is **linear correlation** between  $x$  and  $y$ .  
4 Common sense and care are needed when interpreting scatter diagrams.  
5 Mathematically, there may appear to be a relationship, but this does not imply that  
6 there is a relationship in reality. You might find, for example, that over a period of  
7 time in a particular city there has been an increase in the number of robberies and  
8 an increase in the number of health food shops. It would however be foolish to  
9 imply that there is a relationship between these two variables. The appearance of a  
10 mathematical relationship does not imply that there is a **causal** relationship. An  
11 increase in one variable does not necessarily cause an increase, or decrease, in the  
12 other variable.

### Exercise 2

The text below come from “Design Considerations Upon Product End-Of-Life Options”, L. Barsan (2016), *Journal Of Industrial Design And Engineering Graphics*, Volume 11, Issue 2, November 2016. Perform a similar analysis on this text as the one shown in the example above.

- 1        Increasing the product durability which results in producing less waste can send
- 2        the end-of-life far away, somewhere in the future. Also, globally, less material,
- 3        energy and possible less work is involved. However, the useful life of a product can
- 4        be extended using probably more, better or/and more expensive materials for
- 5        each product unit. These materials might require more expensive or energetic
- 6        intensive technologies, using more complex tools and higher qualified workers.

### Exercise 3

Find a paragraph from a paper of your choice and perform a similar analysis on this text as the one shown in the examples above.

### **Some criteria for a good paragraph**

#### Comment 1

Paragraph building applies to every section you write: abstract, introduction, literature review, methodology, data collection, results and analysis, discussion, conclusion, etc. It also applies to whatever style you write: description, explanation, critique, argumentation, etc. In general we can state four criteria which orient us towards writing a good paragraph:

1. *Use of topic sentences*: We have seen examples of these already;
2. *Adequate development*: Elaborating on an idea to a sufficient degree of detail. We have seen examples of this in our answers to the two exercises above;
3. *Coherence*: This relates to the logical order of sentences which allows the idea to be developed in a unified manner. We have seen examples of this in our answers to the two exercises above.

Paragraphs are incoherent either because we are not using language efficiently or correctly, or because we are trying to link several ideas which are conceptually quite different from each other.

We have seen examples of this in the example and exercise above, where the original collection of sentences were not in any kind of logical or meaningful order.

4. *Unity*: This relates to how the *concept* of one or more ideas are developed. A text which has unity can be seen by the way it clearly links or associates relevant concepts. Writing in such a way clearly show how an idea is being developed. We have seen examples of this in our answers to the two exercises above.

Paragraphs don't have unity when there is too large a jump in the concept we are writing about. In other words, we can't see how two descriptions of an idea are related. The reason this can occur is if i) we are talking about two different ideas in the same paragraph, in which case we should split the paragraph into two separate paragraphs, or ii) we haven't given enough detail, or used language well enough, to make the first idea flow into the (seemingly different) second idea.

It is possible to go against these criteria. We can choose to write paragraphs that "break" some of these rules, but this requires a lot of experience in writing. This is when writing becomes an art rather than just a technique. But, ultimately, such a paragraph still has to read coherently and with unity of ideas in terms of the topic it is trying to expound upon.

Ultimately any segment of text, whether sentence, paragraph, section, etc. has to be traceable back to the underlying idea, concept, theme, and to the thesis of the paper, otherwise such a segment is not relevant to the topic we wish to expound upon.

### Comment 2

By looking at as many examples as possible we can come to see aspects of a paragraph which makes it coherent:

1. It is useful for the first sentence of a paragraph to be a topic sentence;
2. There is certain key vocabulary and phrasing which allows us to develop coherence in the language/discourse of the paragraph;
3. The language of a paragraph should be used to carry and further develop the scientific, mathematical ideas, examples, concepts, etc.

So a paragraph serves to give the reader more substance to an idea of a particular section of the text. In other words, we might say that

**Paragraph building =**

**The development of an idea, with a specific focus and to a sufficient degree of detail, using specific language (such as continuation, transitioning, et.)**

**References**

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